# Soh Cah Toa

Many people learn the rules for right-triangle trigonometry in high school. When I learned the definitions of sine, cosine and tangent, my teacher taught me to remember them like this: *suppose you kick your desk in frustration over your math homework, and hurt your foot in the process -- you should go Soak A Toe (Ahhh!)*

If you know the value of one of the two non-right angles in a right triangle, and the length of one of the sides, you can determine the lengths of the other two sides using

$$\sin\theta = o/h$$
$$\cos\theta = a/h$$
$$\tan\theta = o/a$$
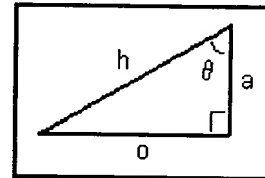
where "o" stands for the side of the triangle opposite to the known angle, "a" stands for the side of the triangle adjacent to the known angle, and "h" stands for the hypotenuse of the triangle.

There are some important rules that will help you to spot or avoid mistakes in calculations with these definitions:

- the adjacent and opposite sides are always shorter in length than the hypotenuse.
- the sine and cosine functions vary in value between zero and one (they are never larger, and come out positive if the angle is considered positive).
- the sine of zero degrees is zero; the sine of ninety degrees is one.
- the cosine of zero degrees is one; the cosine of ninety degrees is zero.

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PROBLEMS - 1

1. A string of length 2L is stretched between two fixed points with a tension T. A force F is applied at mid span normal to the string. The string deflection under the force is x. What is the relation between F and x/L? Assume that T remains constant even though the string is stretching. Also assume that the displacement x is small relative to the string length.

Solution:

Consider equilibrium of the midpoint of the string where the force F is applied. At this point we have three forces acting; the tension in the string on either side of the point and the force F.
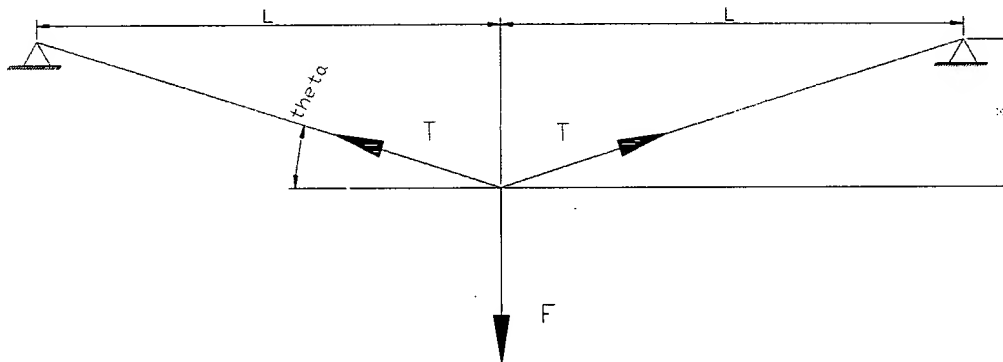
Now, assuming the displacement of the center of the string is small relative to the string length, the angle of the string is simply the slope of the string.

$$\sin \theta \approx \theta \approx \tan \theta \qquad \text{and} \qquad \theta \approx \frac{x}{L}$$

Therefore, equating forces in the vertical direction:

$$F = 2 * T * \sin \theta \qquad \text{or} \qquad F = 2 * T * \frac{x}{L}$$

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From this equation, we notice that the equivalent stiffness of the string at its midpoint is twice the string tension divided by half its length (2T/L) implying a linear force/displacement relationship.

If the displacement x is not small relative to the string length, then:

$$\sin \theta = \frac{x}{\sqrt{x^2 + L^2}}$$ and the force displacement relation is

$$F = \frac{2Tx}{\sqrt{x^2 + L^2}}$$ which is non-linear.

2. A cantilever beam of length L and bending stiffness EI is subjected to a point load F. Determine the relation between F and the deflection at the end of the beam for:
a)   F at the end of the beam
b)   F at the mid point of the beam.

Solution:

a) From beam theory, the bending moment in a beam is related to beam deflection by the following differential equation:

$$M = EI * \frac{d^2 y}{dx^2} \quad (*)$$

and we know that the moment in the beam is given by:

$$M = F * (L - x)$$ where x is the location along the beam from the support.

Integrating this expression twice gives:

$$y(x) = \frac{FLx^2}{2EI} - \frac{Fx^3}{6EI} + Cx + D$$

and the conditions of zero displacement and zero slope at x = 0 imply that both constants C and D are zero.

So the deflection at the end of the beam (x = L) is given by the linear relation:

$$y(L) = \frac{FL^3}{3EI}$$ and the equivalent stiffness of the beam is $$k_{eq} = \frac{3EI}{L^3}$$

b) For the case of the load F located at L/2, the bending moment in the beam becomes:

$$M(x) = F(\frac{L}{2} - x) \qquad \text{for } 0<x<L/2$$

$$M(x) = 0 \qquad \text{for } L/2<x<L$$

Again integrating (*) twice for each of the above expressions gives:

$$y(x) = \frac{FLx^2}{4EI} - \frac{Fx^3}{6EI} \qquad \text{for } 0<x<L/2 \text{ (constants of integration are zero by BC's)}$$

$$y(x) = ax + b \qquad \text{for } L/2<x<L \text{ (where a and b are constants to be determined)}$$

Note that the deflection and slope at the midpoint of the beam are continuous and so we can solve for a and b by matching y(L/2) and y'(L/2) from each of the two above equations.

The results are:

$$a = \frac{FL^2}{8EI} \qquad b = \frac{-FL^3}{48EI}$$

So the deflection at the end of the beam is found to be:

$$y(L) = \frac{FL^3}{8EI} - \frac{FL^3}{48EI} = \frac{5FL^3}{48EI}$$

by plugging a and b into the deflection equation.

The force/displacement relation is thus linear and the equivalent stiffness of the beam is $\quad k_{eq} = \dfrac{48EI}{5L^3}$

3. A simple pendulum consists of a rigid bar, with negligible mass, of length L, with a point mass m. The displacement is measured by the angle θ measured relative to the vertical. The effect of gravity is to produce a moment M about its pivot that rotates the pendulum. What is the relation between M and θ?

Solution:

As the pendulum is in equilibrium, the moment about the pivot due to gravity acting on the mass must be equal and opposite to the moment at the pendulum pivot.
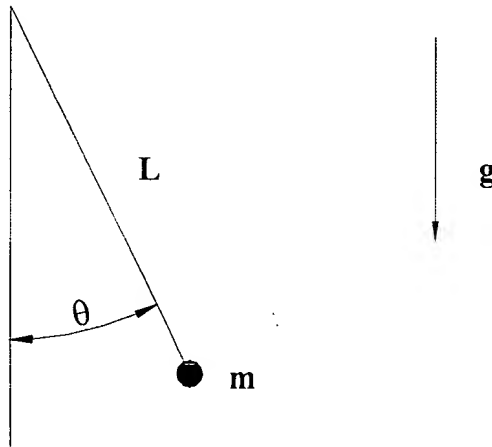
$$M = mgL\sin\theta$$

Note that sinθ can be represented by the series:

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

and for θ (in radians) < about 1, sinθ is closely approximated by θ so for small displacements, the applied moment is related to the displacement by:

$$M = mgL\theta$$

For example, sin(0.5 rad ~ 6°) is approximately 0.48 (very close to 0.5).